# Negative refraction in two-dimensional photonic crystals*

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**Abstract** Recent progress made in research on negative refraction in two-dimensional photonic crystals is reviewed. Special attentions are paid to the research results of our group. We explore the underlying mechanism governing the occurrence of negative refraction in periodic and quasiperiodic photonic crystals. These results may be useful references for the application of photonic crystals. The perspectives of future research on negative refraction are briefly predicted.

**Keywords:** **negative refraction, photonic crystals, superlens, subwavelength imaging, equifrequency surface.**

Photonic crystals (PhC), composites with periodic arrangement of dielectric or metallic materials, possess photonic band gap (PBG) due to the periodic modulation of dielectric constant. PhC exhibit a wide variety of physical phenomena[1—5]. Many applications[6,7] have already been proposed and demonstrated. Research on PhC has been a flourishing area during the past twenty years. However, most of the PhC applications rely on the presence of PBG. Recently, new physical phenomena in PhC working outside the band gap[8,9], such as negative refraction[10] that means a ray incident on the interface of such a material would refract on the same side of the normal, have caught much attention. Negative refraction has a large number of potential applications in different fields such as optics, materials science, biology, and biophysics, which has instigated an extensive research in the area of negative index materials.

Negative refraction (NR) was discussed by Veselago early in the 1960s[11]. He considered the optical properties of an imaginary material with negative dielectric permittivity $\varepsilon$ and magnetic permeability $\mu$. In his materials, the vectors $\boldsymbol{E}$, $\boldsymbol{H}$ and $\boldsymbol{k}$ form a left-handed triplet, instead of a right-handed triplet occurring in conventional media. This new kind of materials was called left-handed materials (LHM) or negative index materials (NIM). LHM exhibit peculiar physical properties, including negative refraction, inversed Snell's law, reversal of Doppler shift, and backward Cerenkov radiation. One of the important properties is negative refraction, which appears when the electromagnetic wave with certain resonant frequency propagates through the left-handed material[12—14]. Because there are no naturally occurring materials with both negative $\varepsilon$ and negative $\mu$ on which to test his hypothesis, this subject was only a theoretical curiosity until the 1990s. Pendry proposed the designs to realize negative $\varepsilon$ and negative $\mu$[15,16] and pointed out that negative refraction could make a perfect lens capable of subwavelength imaging[17].

In addition, the concept that PhC can exhibit negative refraction has received recognition. Many schemes have been proposed to study the optical properties of PhC exhibiting negative refractive effect, particularly the superlens imaging. Results show that all-angle negative refraction can lead to superlensing[18], and the image resolution can be improved by choosing appropriate surface termination[19] or by employing a complex basis[20] at fixed frequencies or by optimizing the air hole radius and the air hole shape[21]. Two-dimensional (2D) PhC have attracted much attention due to their amenably controlled fabrication. The negative refraction in three-dimensional (3D) PhC has been predicted theoretically[22,23] and demonstrated experimentally[24,25]. In this review, we will not attempt to mention all the fields relevant to the study of negative refraction. Based on our group's research, we will only review the recent development in negative refraction in 2D photonic crystals. The right-handed and left-handed cases are dis-

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cussed in detail to clarify the physical origins of the negative refraction in PhC.

## 1 The basis of theoretical analysis-wave vector diagram

Photonic crystals can be regarded as the optical analogy of conventional crystals. The dielectric "potential" may produce the same phenomena for photons as the atomic potential does for electrons. This means that the concepts of reciprocal space, Brillouin zones (BZ), dispersion relations, Bloch wave functions, and Van Hove singularities, etc., must be applied to photon waves. The well-established routine towards the investigation of negative refraction and focusing effect starts from the analyses of the photonic band structure and the equifrequency surface (EFS)[10,18,26—40].

EFS plot is contour diagram in the wave vector space for a constant frequency. In PhC, the shape of the EFS is frequency dependent. The group velocity vector is defined by $\boldsymbol{v}_g = \nabla_k(\omega(\boldsymbol{k}))$. The normal to the EFS gives the group velocity vector direction, indicating the energy flow Poynting vector direction in PhC. The conservation of tangential component of wave vector at the interface between two materials is always satisfied. Under these conditions, the direction of light propagation inside the PhC can be obtained. The steps to determine the refraction direction were described in detail in Ref. [40]. In this paper, we study the refractive phenomena occurring in PhC in terms of wave vector diagram formalism.

## 2 Negative refraction in periodic crystals

Theoretical studies indicate that the negative refraction of electromagnetic (EM) waves in PhC is not unique. One is the left-handed behavior[10,26—30], where the group velocity and the phase velocity derived from the band dispersion are antiparallel to each other for all the values of wave vector $\boldsymbol{k}$ (i.e. $\boldsymbol{S}\cdot\boldsymbol{k}<0$, where $\boldsymbol{S}$ is the Poynting vectors), leading to $n_{\text{eff}}<0$ for the PhC. This has been demonstrated experimentally[29,30]. However, negative refraction has also been studied in another case[18,31—35]. Negative refraction may occur when the incident field couples to a band with convex EFS contours in the $\boldsymbol{k}$-space. In this case, neither the group velocity nor the effective refractive index is negative and PhC behaves much like a uniform right-handed medium (i.e. $\boldsymbol{S}\cdot\boldsymbol{k}>0$) but exhibits negative refraction. This case has also been demonstrated experimentally[34,35]. We will discuss them in the following sections. Plane-wave expansion method (PWEM) is used to obtain the band structure and EFS contours. Another method, multiple-scattering Korringa-Kohn-Rostoker[41], is also adopted to calculate the EFS contours for metal case. To investigate the field distribution of EM wave propagation, we employ the finite difference time domain (FDTD) method with perfectly-matched layer (PML) boundary condition.

### 2.1 The right-handed behavior

The working frequency is located at the lowest photonic band of the PhC made of a triangular lattice of air holes embedded in a dielectric background[36]. Both the TM and TE polarized EM waves are investigated. Here, we illustrate with TM mode.

Fig. 1 displays the EFS at several relevant frequencies for the TM-polarized Bloch modes within the first Brillouin zone (BZ). The EFS contours of some frequencies such as 0.05, 0.1, 0.15 (in the unit of normalized frequency $2\pi c/a$, where $c$ is the light speed in vacuum, and $a$ is the lattice constant) are close to a perfect circle, indicating that the crystal behaves like an effective homogeneous medium at these long wavelengths. However, the 0.175 contour is significantly distorted from a circle. Furthermore, the EFS contours of this polarization mode within the first band are convex at certain frequency windows. Fig. 2 shows the $E_z$ field distribution across the PhC slab for TM polarization simulated by the FDTD method. The operating frequency is set at $\omega = 0.179$ $(2\pi c/a)$. The surface normal of the flat PhC lens is along the $\Gamma K$ direction. Clearly a high-quality image is formed on the opposite side of the flat slab. Note that the image spot is located in the vicinity of the slab.

According to the same analysis as the above, we

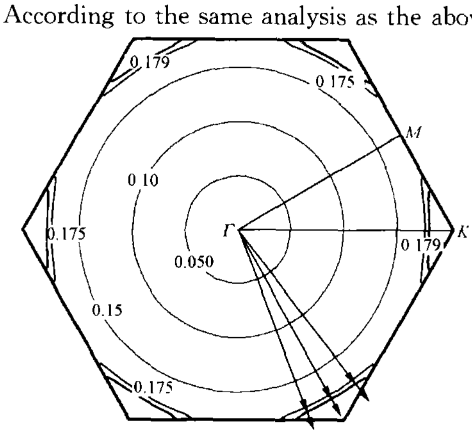


Fig. 1. Several EFS contours of the first band at the TM mode in the first Brillouin zone. The permittivity of background dielectric is $\epsilon = 12.56$. The air hole radius is $r = 0.15a$.

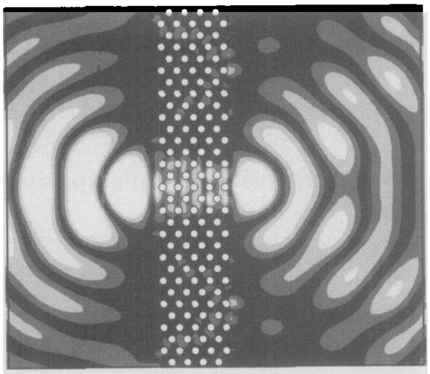

Fig. 2. Simulated $E_z$ field distribution of a point source $\omega = 0.179(2\pi c/a)$ located at $a$ away from the 9-layer air-hole slab for TM polarization.

theoretically studied the imaging properties of PhC lens consisting of a square lattice of rectangular[37] and elliptical[38] dielectric rods in air. A good-quality image can also be formed against the slab within a frequency window at the fundamental band.

In the above cases, images are all strongly confined to the near-field region independent of the slab thickness. The imaging behavior clearly does not obey the well-known wave beam refraction law. In other words, the formation of an image is dominantly governed by the self-collimation effect[32] although the negative refraction effect might play some roles in imaging.

In addition, an interesting phenomenon of image downshift is observed if we change the orientation of the rectangular rods. The geometry of such PhC slab is depicted in Fig. 3. The rods are arranged asymmetrically with respect to the surface normal of the slab. Fig. 4 displays the vertical shift of the image

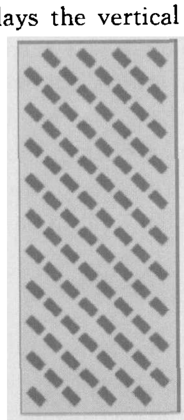

Fig. 3. The schematic diagram of rectangular rods in a PhC slab with the surface normal along the $\Gamma M$ direction. The long side of the rods is along the $\Gamma X$ direction. The length and width of the rectangular rods are $l_1 = 0.75a$, and $l_2 = 0.375a$, respectively. The dielectric constant of the rectangular rods is $\epsilon = 14$.

spot away from the horizontal line passing the point source. The similar shift occurs in the elliptical rods case if the major or minor axis of the elliptical rods is not collinear with the surface normal. This image shift is impossible in a PhC structure made up of isotropic circular rods. The strong geometric anisotropy inherent in a rectangular or elliptical rod is responsible for the shift, which offers additional probability to control the focus of PhC slabs.

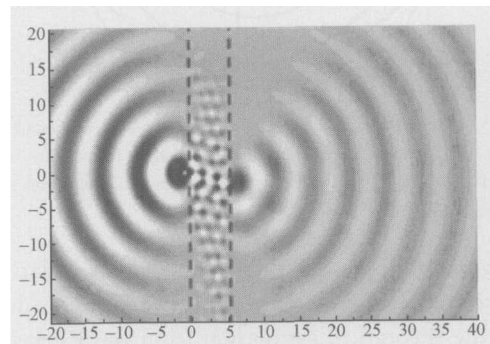


Fig. 4. $E_z$ field patterns of a point source $\omega = 0.196(2\pi c/a)$ placed at $x = -a$, $y = 0$. The rectangular-rod PhC slab is 8 layers thick and 60 layers wide, whose surfaces are represented by two dashed lines.

## 2.2 The left-handed behavior

We focus the analyses on the wave propagation in the second band. The PhC consist of a triangular lattice of coated cylinders immersed in a Styrofoam template[39]. The cylinders have copper cores coated with dielectric material, which have been investigated theoretically in Ref. [28].

The EFS contours of PhC at several relevant frequencies are shown in Fig. 5. It is clear that when frequency increases, the EFS moves inwards. The Poynting vector $\boldsymbol{S}$ is anti-parallel to the wave vector $\boldsymbol{k}$, which indicates that this PhC has a left-handed behavior. In other words, the direction of phase velocity is opposite to the direction of the group velocity and energy propagation. This corresponds to the fact that the PhC possess a negative effective refractive index $n$ (NRI) related to the phase velocity according to $v_p = c/n$. From the EFS, $n_{\text{eff}} = -1$ is expected at 12.7 GHz, which is close to the ideal LHM system.

The refraction experiment is performed on different wedge shaped samples of different wedge angles in a semicircular cavity. By examining the refraction angle $\theta$ varying with the incident angle $\theta_0$, we obtain the refractive index. We find that at 12.7 GHz, $\theta$ is linearly proportional to $\theta_0$ in the whole an-

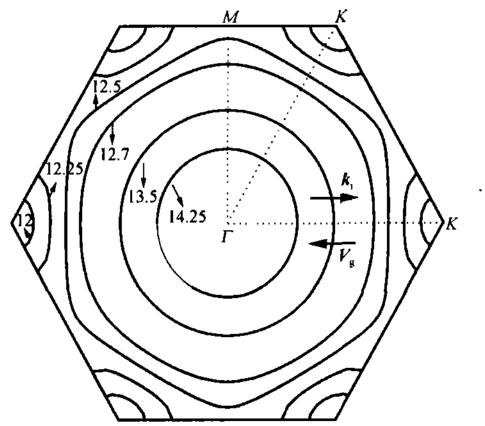


Fig. 5. Several EFS contours for S wave of the second band within the first Brillouin zone. The numbers denote frequencies in unit of GHZ. $\boldsymbol{k}_i$ and $\boldsymbol{V}_g$ represent the wave vector and the group velocity, respectively.

gle region. That is, at this frequency the relative refractive index is −1. The experimental result confirms the theoretical estimation from the EFS.

In order to clarify the dependence of imaging on the slab thickness and the object distance, we conduct an experiment in which a monopole antenna is used as the point source. The power distribution is measured by scanning the transmission intensity across the focal point and along the surface normal of the slab. Our experimental observations indicate clearly that $U + V = L$, where $L$ is the thickness of the slab. $U$, $V$ are the source- and image-distances, respectively. In contrast to the images in the near-field region for the lowest valence band, the well-known wave-beam negative refraction law has been demonstrated in this case.

### 2.3 Summary

The optical properties of PhC exhibiting negative refraction are studied for the right-handed and left-handed cases, which seems rather different at first glance. However, more careful analysis shows that they can be interpreted by the same principle. In the left-handed case, the maximum of the dispersion curve is located at the $\Gamma$ point of the Brillouin zone. As for the right-handed case, they have the local maximum located at a $k$-point away from the $\Gamma$ point (another high symmetry point in the BZ, e.g. $M$ point in square lattice or $K$ point in triangular lattice). This means that the photonic effective mass near a local maximum of the dispersion curve is negative-definite. In the sense of the relative wave vector, they share one common feature: $\boldsymbol{q} \cdot \partial w / \partial \boldsymbol{q} < 0$, where $\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{K}_0$ is the relative wave vector, and $\boldsymbol{K}_0$ is the wave vector of the local maximum, $\partial w / \partial q$ is the group velocity. This means that the relative wave vector of a local frequency maximum is pointed opposite to the group velocity.

## 3 Negative refraction in quasiperiodic photonic crystals

The photonic quasicrystals (PQC) considered here are a random square-triangle tiling system possessing 12-fold symmetry. Feng has pointed out that negative refraction exists in photonic quasicrystals (PQC)[42]. The multiple-scattering method is used to calculate the propagation of EM waves in PQC[43]. It is found that not only images are formed against the slab, but also the subwavelength resolution below the conventional diffraction limit is achieved. A series of slab samples with different thicknesses are adopted and the images can exist beyond the near-field region. The power distribution along the transverse and longitudinal directions across the image point was measured. The measured results are consistent with the calculated results.

It's an intriguing phenomenon for negative refraction occurring in PQC. The physical principles for NR in the periodic PhC arise from the dispersion characteristics of wave propagation in a periodic medium. NR can be analyzed by using EFS of the band structures as described above. However, what is responsible for the negative refraction in PQC? It was suggested that analogous concepts of Bloch functions and Bloch-like states in the periodic structures could be applied to some quasicrystals[44]. Thus, the origin of the negative refraction in the PQC can be understood similar to that in the periodic PhC. It is convinced that the triangle and square unit, when arranged in a periodic lattice, can show negative refraction. If we bring them together into a quasicrystal, the negative refraction phenomenon is still expected. The experiments verified our intuition. PQC can be another choice to achieve negative refraction.

## 4 Perspectives

Great progress in negative refraction of PhC has been achieved in the past few years. However, at present, there is still a challenge. Most of practical applications of these unique NIMs are in the optical and near-infrared parts of spectra. It is of particular importance to integrate the negative index imaging into compact optical circuits. So far, experimental and theoretical studies are conducted mainly in the microwave region and there are few reports on near-infrared wavelengths[45,46]. A major goal of this field of research is the development of media with negative refractive indices that can be scaled to THz and optical frequencies[47,48]. In a word, negative refraction is expected to be a significant step towards novel imaging optics and can lead to considerable changes in optical system design. Negative index materials will play an important role in the future.

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